Self-stabilized fractality of sea-coasts through damped erosion

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Plan of the talk

- Motivations
- Coastal dynamics
- Modeling the sea:
  - Fractal acoustic cavities
- Modeling the earth
- Erosion dynamics
- Real data analysis
  - Earth coastlines
  - Mars coastlines?

Motivations

Coastal systems

"The coast is simply where the land meets the sea."

from *Coastal Systems* by Simon K. Haslett
(Routledge, Taylor and Francis, London 2000)

"As a matter of some urgency, researchers concerned with coastal evolution should consider the alternative models, even if there are few supporting data. The ideas of non-linear response, stochastic developement, deterministic chaos, catastrophism and criticality all deserve investigation."

from "Coastal Evolution. Late Quaternary shoreline morphodynamics" Ed. R.W.G.Carter and C.D. Woodroffe
(Cambridge University Press 1994.)

Rocky Coasts

Rocky coasts are estimated to represent 75% of the world's shorelines, but this includes beaches backed by rocks, with many different morphologies and several different dynamical processes in action. Nevertheless, there are many cases in which wave ***erosion*** is recognized as the main erosive process.

- Collision coasts tend to be rocky containing few depositional features.
- Tectonically active coasts often display rocky coasts with very limited sediment deposited by rivers

One can think that wave erosion can play a role in relatively low rocky coasts. Instead, the height of the cliff is not a general contraindication for a sea erosive dynamics.

Figure 3-8. Southeast coast of Hawaii, near Kalapana. Rugged cliffs are built up of numerous lava flows.

from: *Beaches and Coasts* by R. A. Davis Jr. and D. M. Fitzgerald, Blackwell Publishing, Oxford, 2004

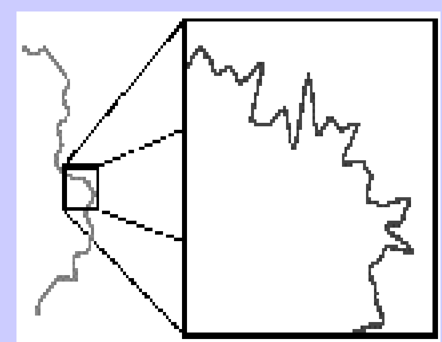
Ad nauseam

Rocky coasts are often rugged, and have been taken as an introductory archetype of fractal morphology in nature.

Mandelbrot B. B.

*How long is the coast of Britain? Statistical self-similarity and fractional dimension***,** Science, **155**, 636 *(1967)*

$$L \sim L_0^{1-D_f}$$



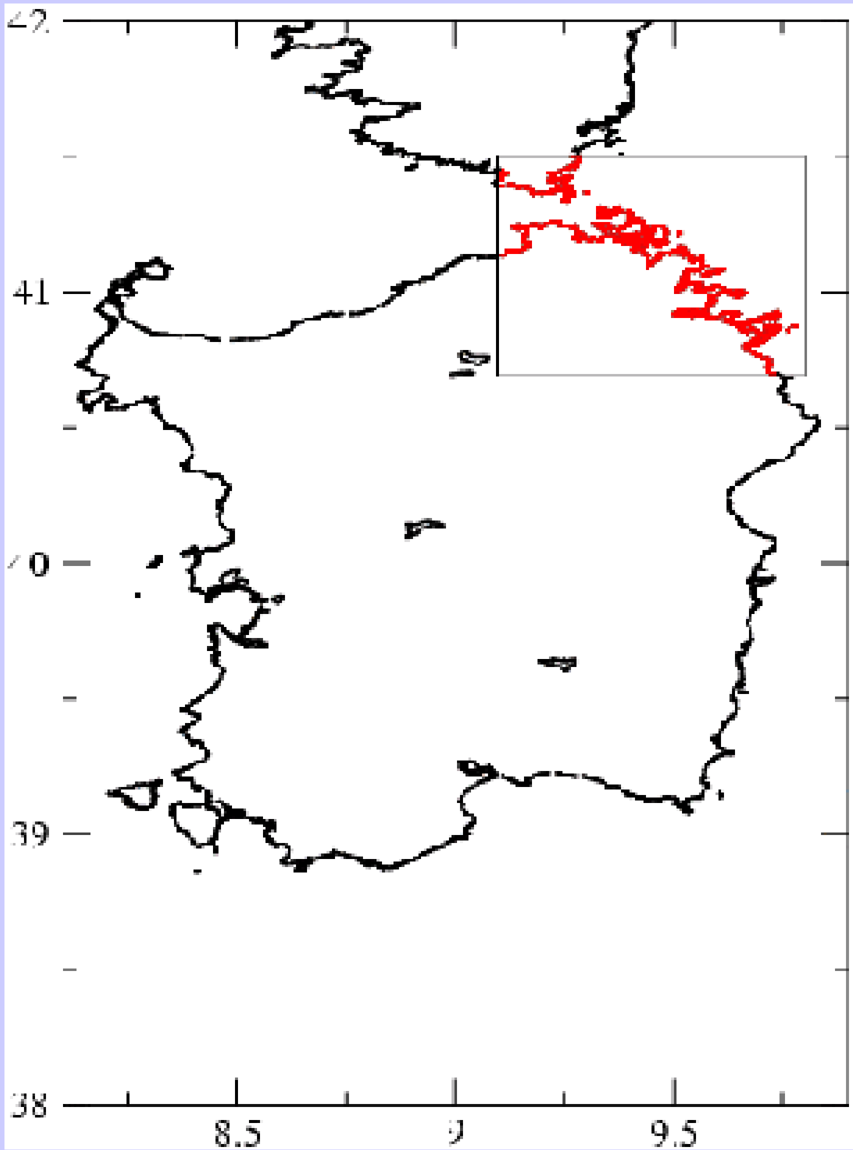
pictures from:
`http://www.math.vt.edu/people/hoggard/FracGeomReport/node1.html`

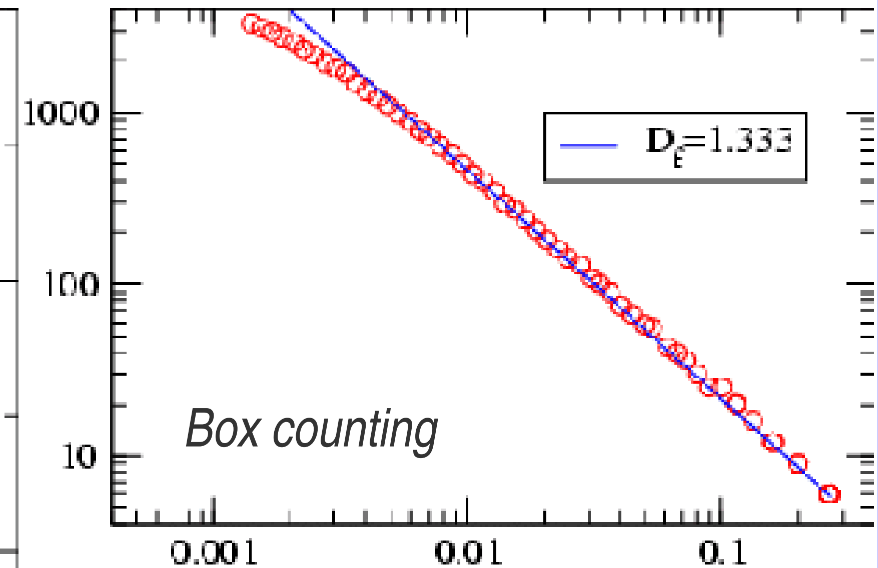
International workshop for the 80th birthday of B.B.Mandelbrot: **http://aeis.ifrance.com/aeis/**

Why?

Sardegna, Italy

An example of rugged coast in Italy.





Data from The Coastline Extractor: http://rimmer.ngdc.noaa.gov/coast/getcoast.html

The model

Erosion

Different processes and time scales

“Rapid:”

- wave quarrying

“Slow:”

- abrasion
- frost shattering
- thermal expansion
- salt water corrosion
- carbonation
- hydrolysis
- …

from: Davis R. A. *Oceanography-An Introduction to the Marine Environment*,
W.C. Brown publ. Dubuque, Iowa, 1986.

Coastlines damping feature

"Islands":

"Fjiords":

Coastlines damping feature

“Screening”:

A minimal model

Our minimal model for rocky coast formation bears on the reciprocal evolution of the erosion power and the topography of the coast submitted to that erosion:

The more irregular the coast, the weaker the sea-waves and the sea erosion power.

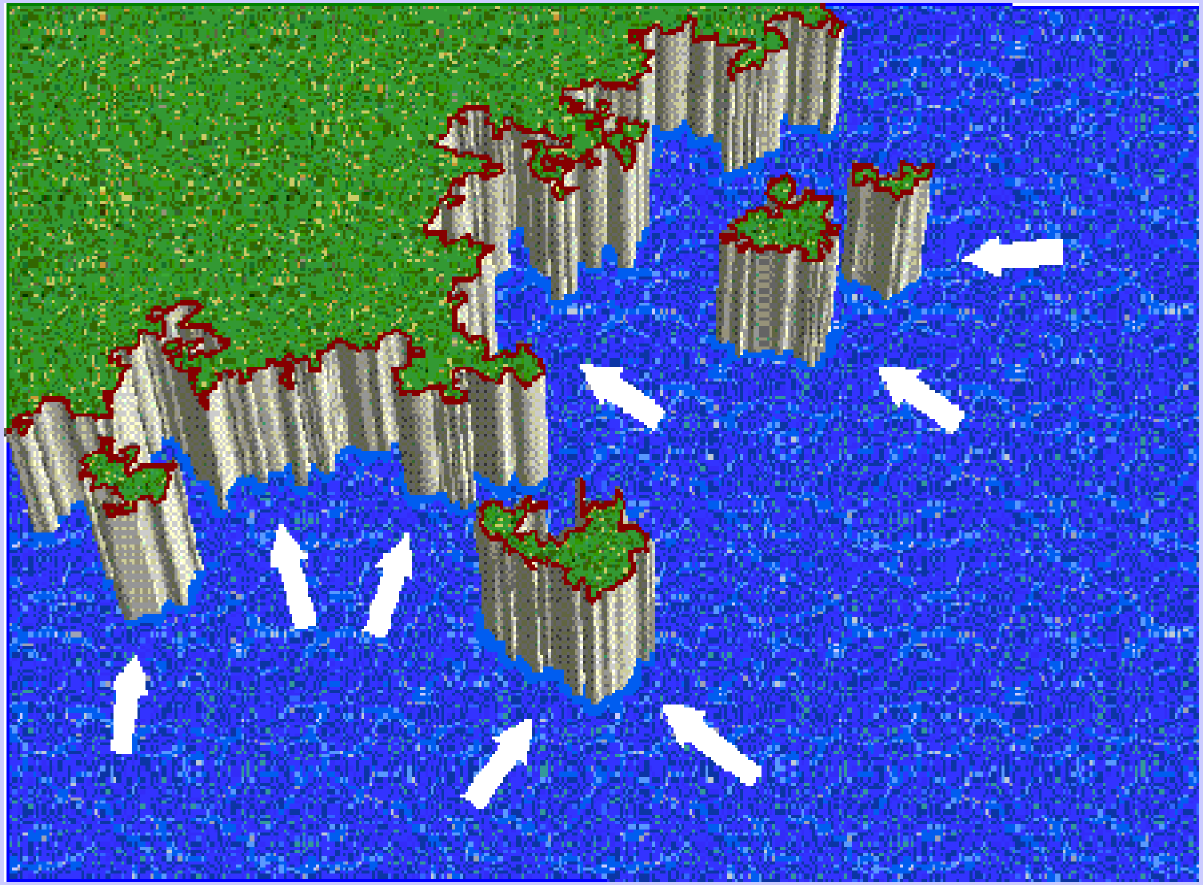
This retroaction leads to a fractal sea-coast with

$$D_f = 4/3$$

(gradient percolation universality class).

The fractal geometry plays the role of a morphological attractor:

This model reproduces at least qualitatively some of the fundamental features of real coasts using only simple ingredients.



Model limitations

- No transport of sediments (they are supposed to be transported off-shore)
- No clear space-time scale to relate with dynamics of real coastlines

Model universality

Simplifications that keep the system in the universality class of percolation are unimportant:

- Randomness of lithology, small scale correlations, slow weakening rule, more realistic damping model...

Fractal geometry of large scale coastline morphology is the result of a (critical) self stabilization mechanism between erosion and damping.

Modeling the sea

The sea, with the coast, is considered as a resonator.

- There exists a given average power of the waves P_0.

- The "force" ***f(t)*** acting on the unitary length of the coast is proportional to P_0, their ratio is *the quality factor Q*, measuring damping:

the smaller the quality factor, the stronger the damping of the sea-waves.

- Studies of fractal acoustical cavities show that viscous damping increases roughly proportionally to the perimeter of the cavity

(a mathematical fractal cavity should be totally damped!).

- Therefore, *in first approximation*,

the contribute to the quality factor Q due to coast morphology is proportional to the inverse of the coast perimeter length $L_p(t)$.

$$f(t) \sim 1/L_p(t)$$

The precise law is not so important

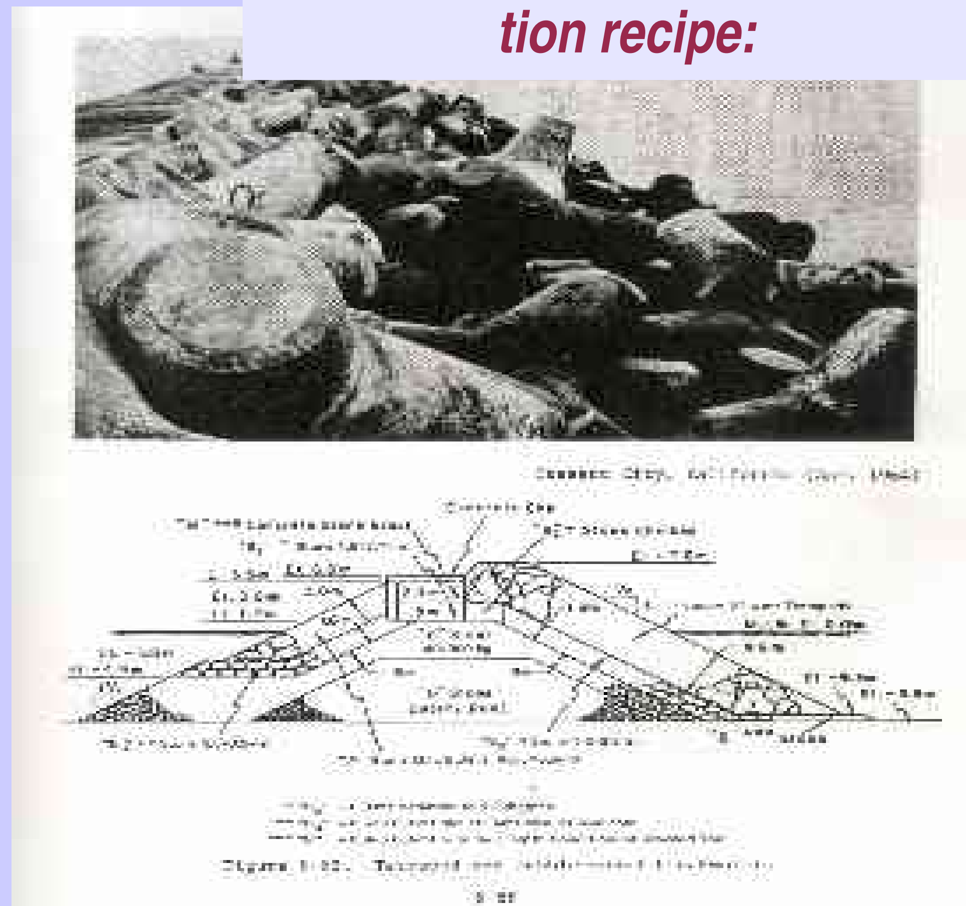
Fractal acoustic cavities

Experimentalists know that "good" resonators have generally smooth or regular geometry …
A **"good"** resonator stores reactive (or kinetic) energy with little dissipation.
A "**bad**" resonator dissipates rapidly the reactive (or kinetic) embedded in it.

What about irregular or even fractal resonators?

Absorbing Acoustic Barrier (patented by BS)

Empirical breakwater construction recipe:



Modeling the earth

- The "resisting" earth is represented by a square lattice
- *Random lithology:* A random number $\boldsymbol{x_i}$ is assigned to each earth site *i*
- The resistance to the sea $\boldsymbol{r_i}$ depends on lithology and local environment

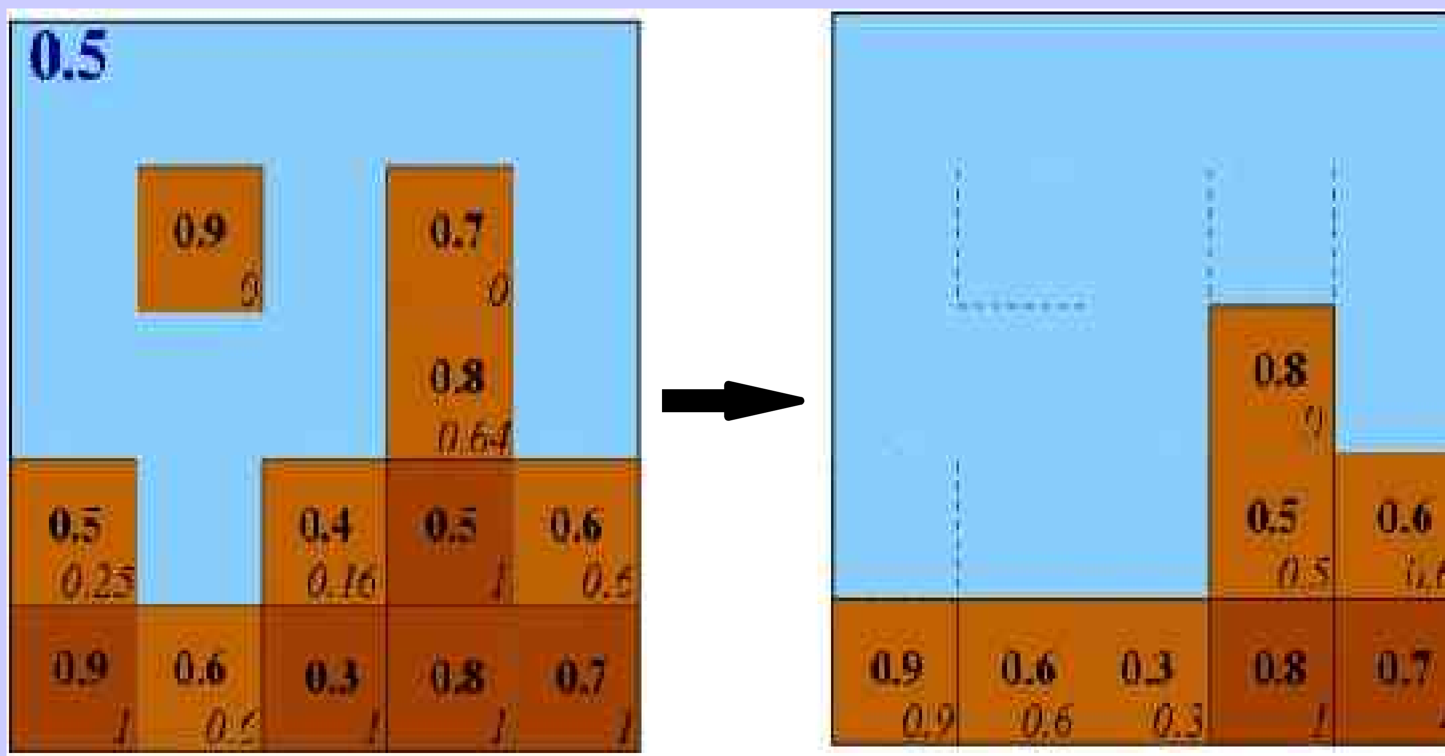
(a site surrounded by the sea it is submitted to a larger erosion)

Rule:

$r_i = x_i$ if only 1 n.n. is occupied by the sea.

$r_i = x_i^2$ if 2 n.n. are occupied by the sea

$r_i = 0$ if more than 2 n.n. are occupied by the sea

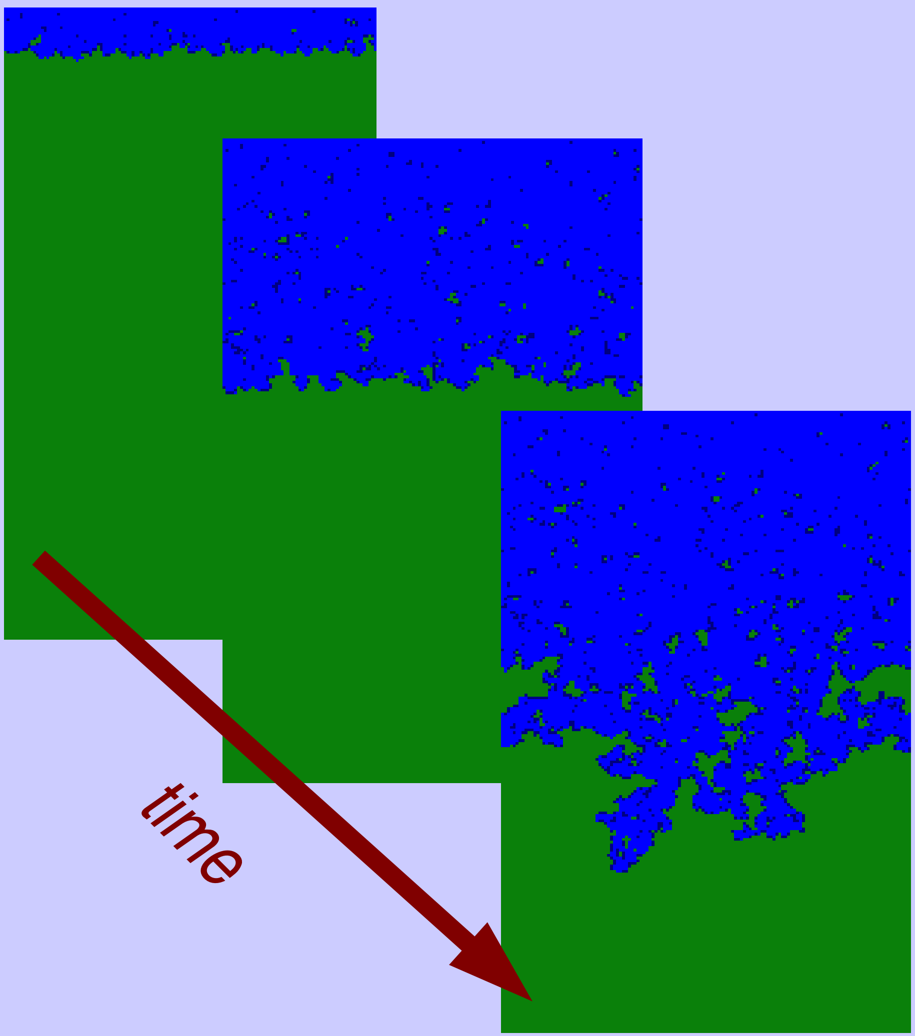


time=t ***time=t +1***

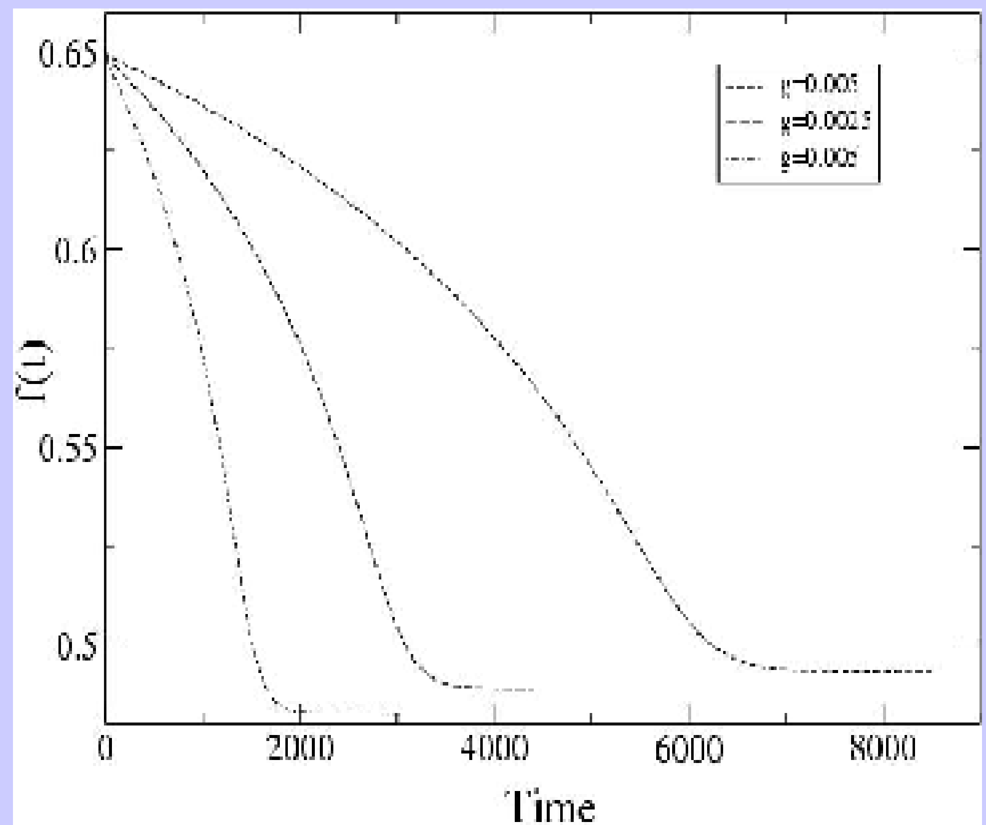
Model dynamics (I)

Fast erosion process

- The sea erodes the coast;
- Coast becomes irregular, damping futher sea erosion;
- Sea erosion spontaneously stops on a “hard” coastline.



Sea erosion force



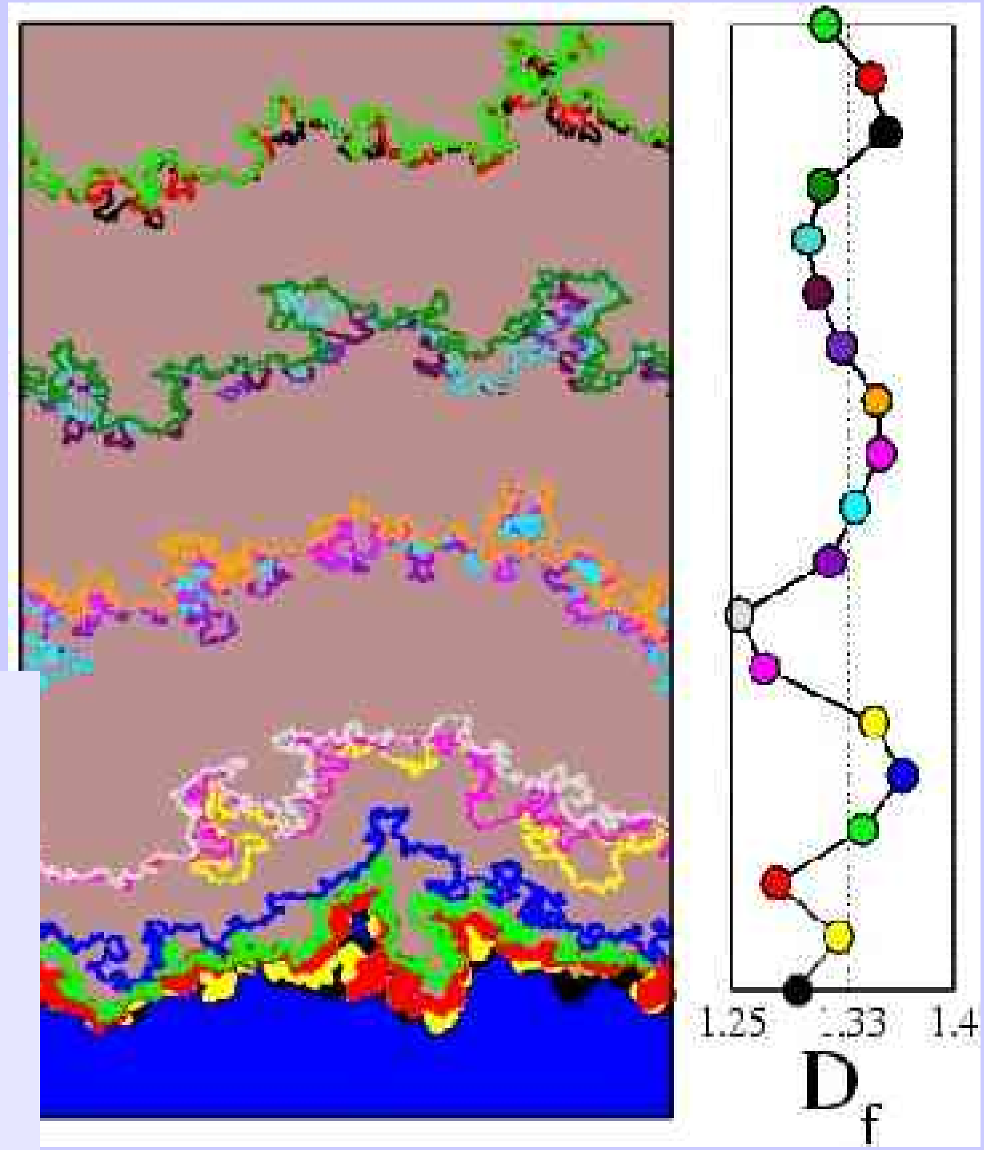
Model dynamics (II)

Slow erosion

- Lithology parameter x_i of the coast sites is slowly decreased until a site becomes weaker than sea erosion force
- Rapid sea erosion starts again ("avalanches" dynamics)

Complete dynamics

fast+slow erosion`



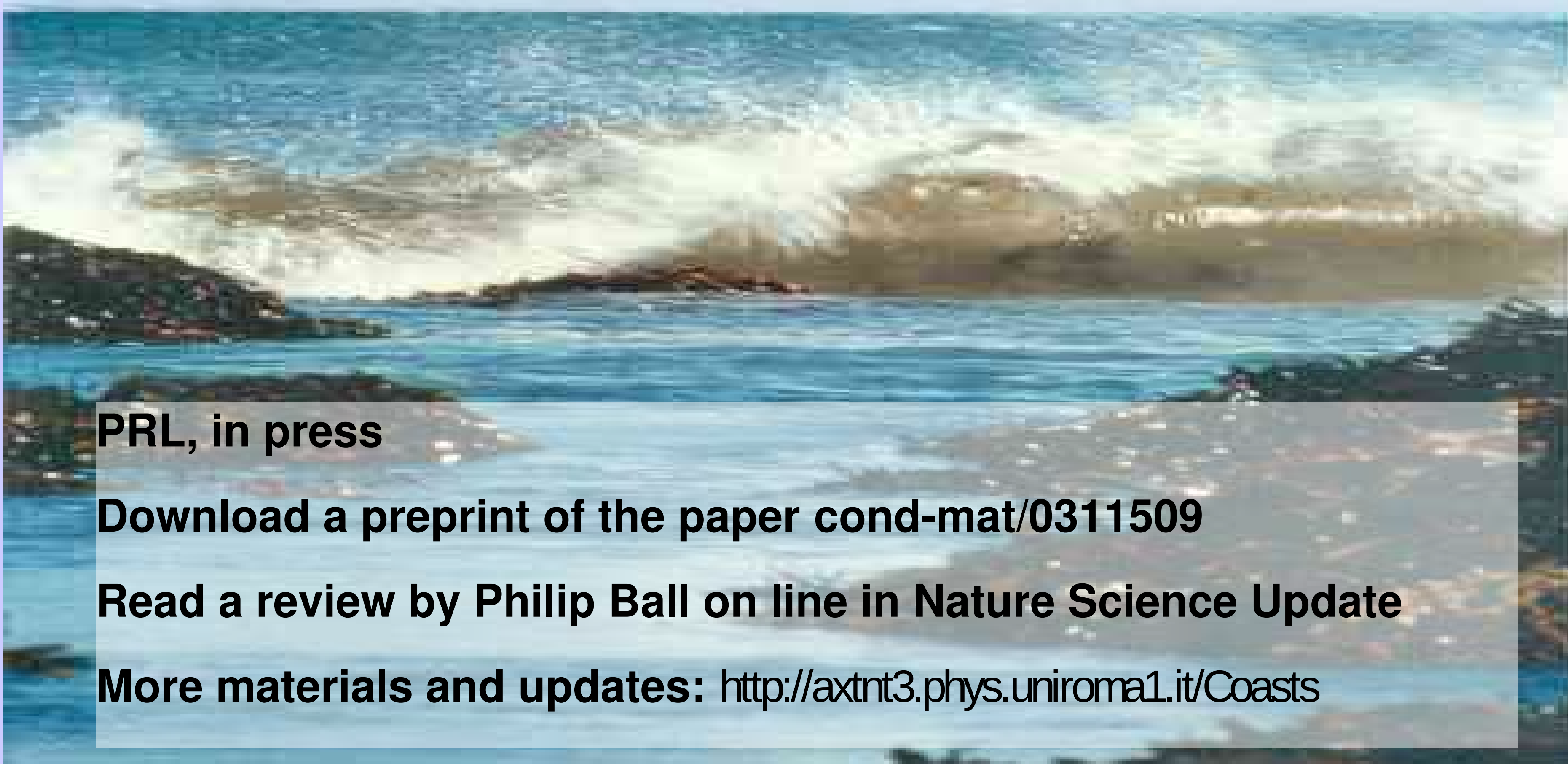
Successive colors represent different time steps with the ***same*** weathering deterioration.

The measured fractal dimension fluc-tuates around 4/3.

Self-Stabilised Fractality of Sea-Coasts through Erosion

by Bernard Sapoval, Andrea Baldassarri, Andrea Gabrielli

We propose a minimal model for the formation of rocky coast morphology. This model bears on the reciprocal evolution of the erosion power and the topography of the coast submitted to that erosion: The more irregularly eroded the coast is, the weaker the average sea erosion power. This retroaction leads to the spontaneous formation of a "stable" fractal seacoast.



PRL, in press

Download a preprint of the paper cond-mat/0311509

Read a review by Philip Ball on line in Nature Science Update

More materials and updates: http://axtnt3.phys.uniroma1.it/Coasts

Real Data Analysis

Real data analysis

Using the data available from the GSHHS Database, A Global Self-consistent, Hierarchical, High-resolution Shoreline Database (created and maintened by Paul Wessel) we have computed the "local" fractal dimension of the whole world coastlines. To this aim we have considered a grid of points separated by 1 degree of latitude/longitude. At each grid point we have associated the fractal dimension (computed via box counting) of the coastlines contained in a squared angular region of 2 degrees of latitude/longitude side centered on the point.

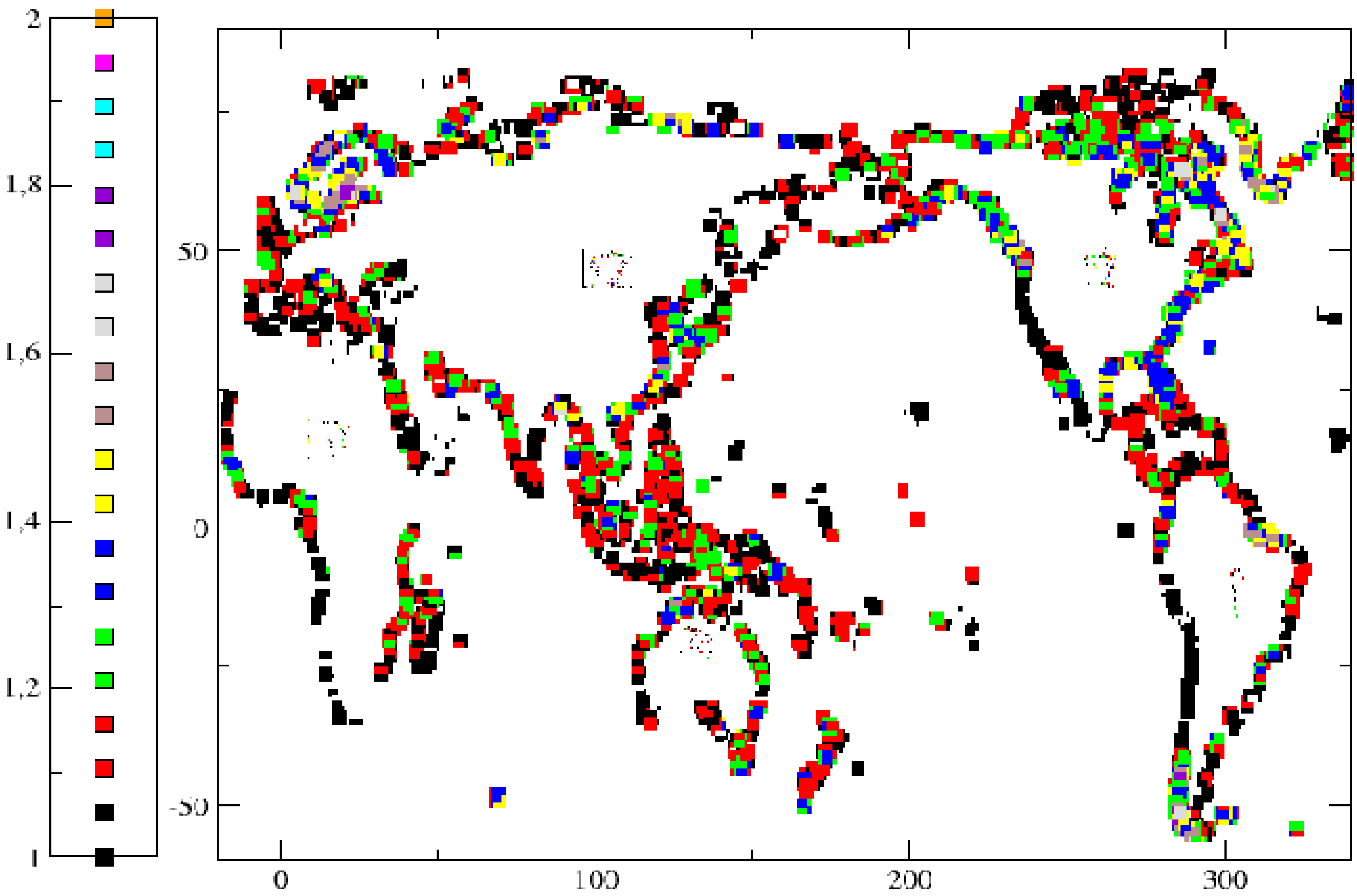
GSHHS Database

http://www.soest.hawaii.edu/wessel/gshhs/gshhs.html

See also The Coastline Extractor:

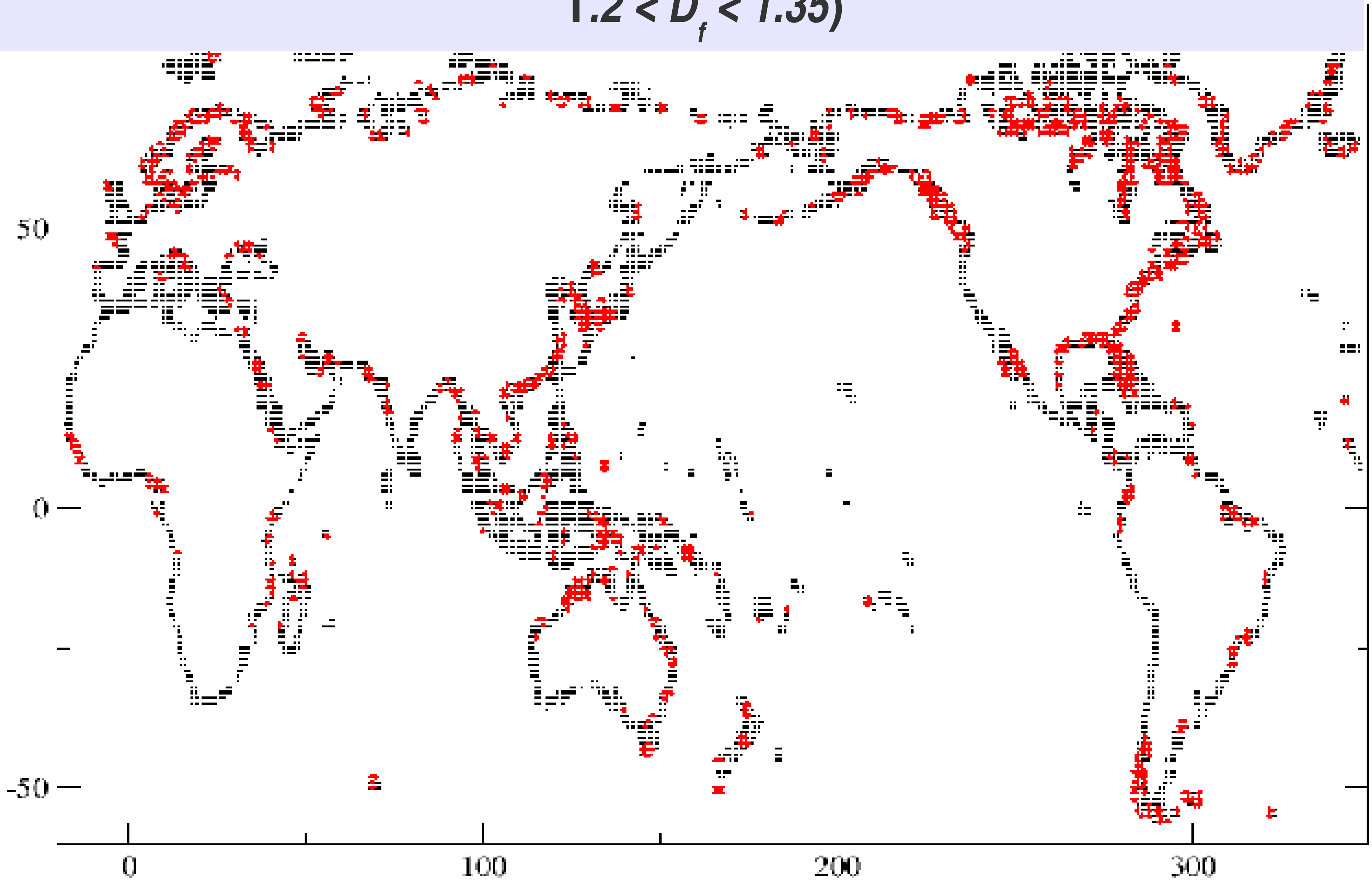
http://rimmer.ngdc.noaa.gov/coast/getcoast.html

Fractal analysis of World coastlines



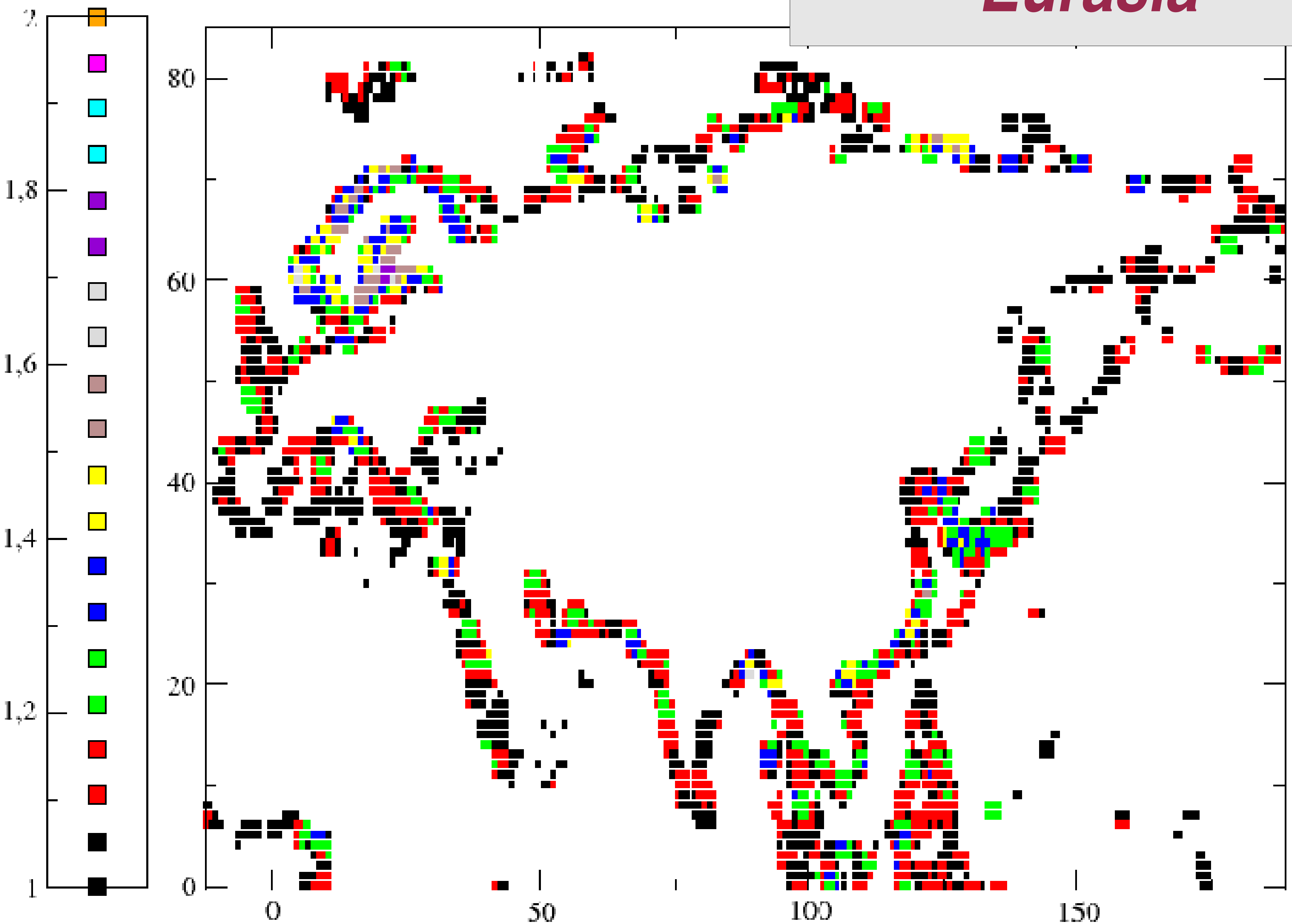
Fractal coasts in the world

Fractal coasts with measured fractal dimension around *4/3* ($1.2 < D_f < 1.35$)

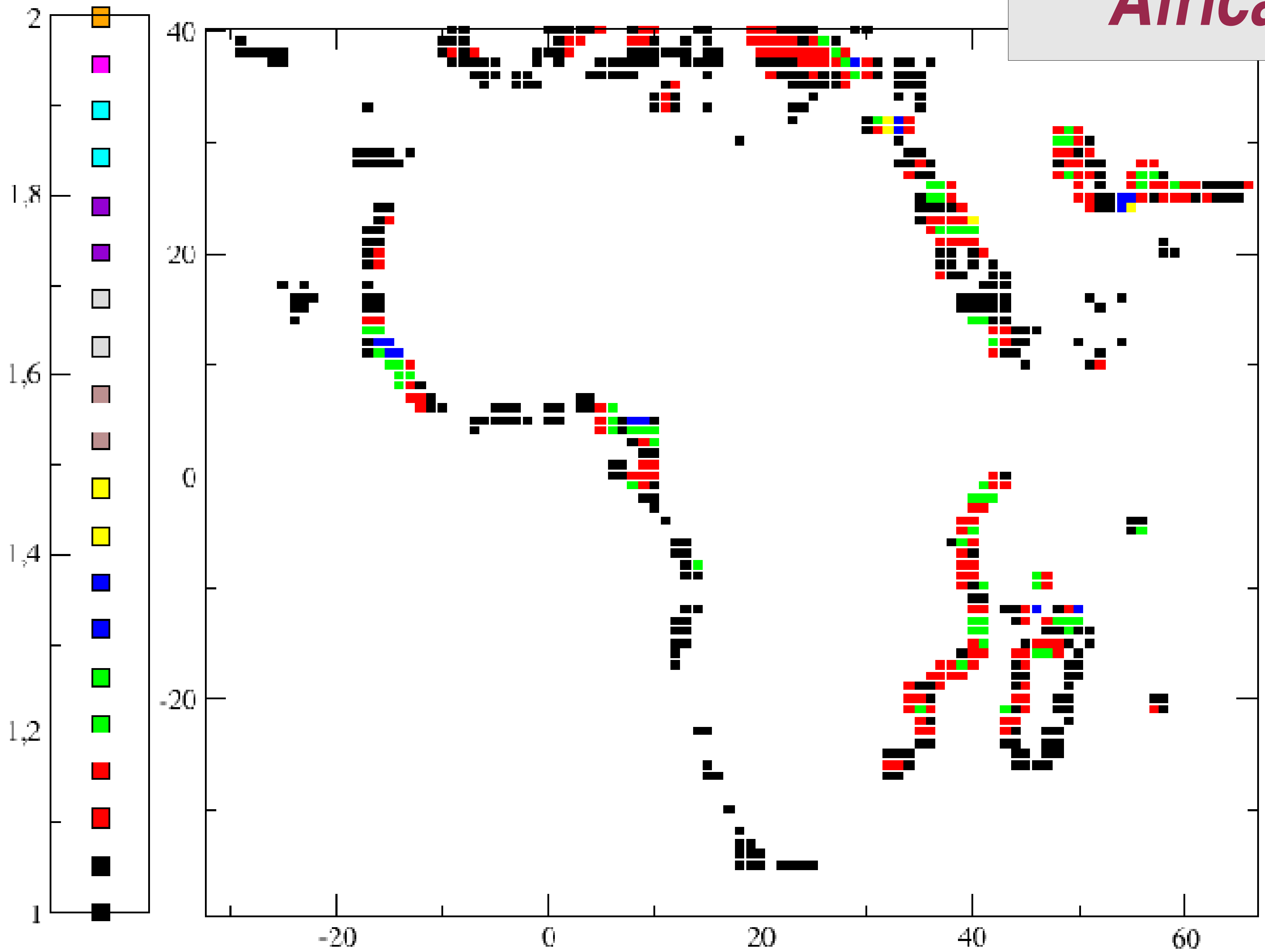


Additional Materials

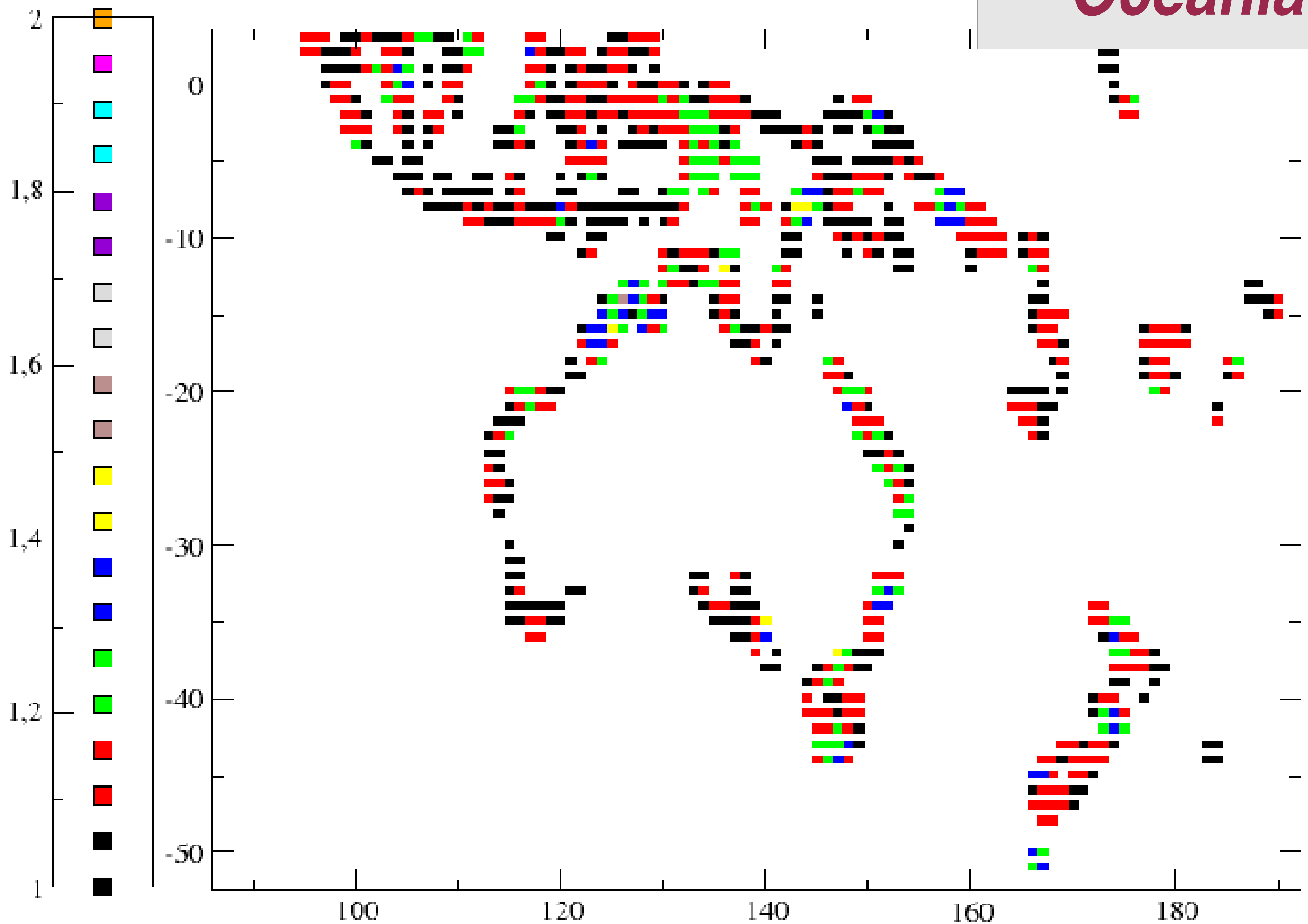
Eurasia



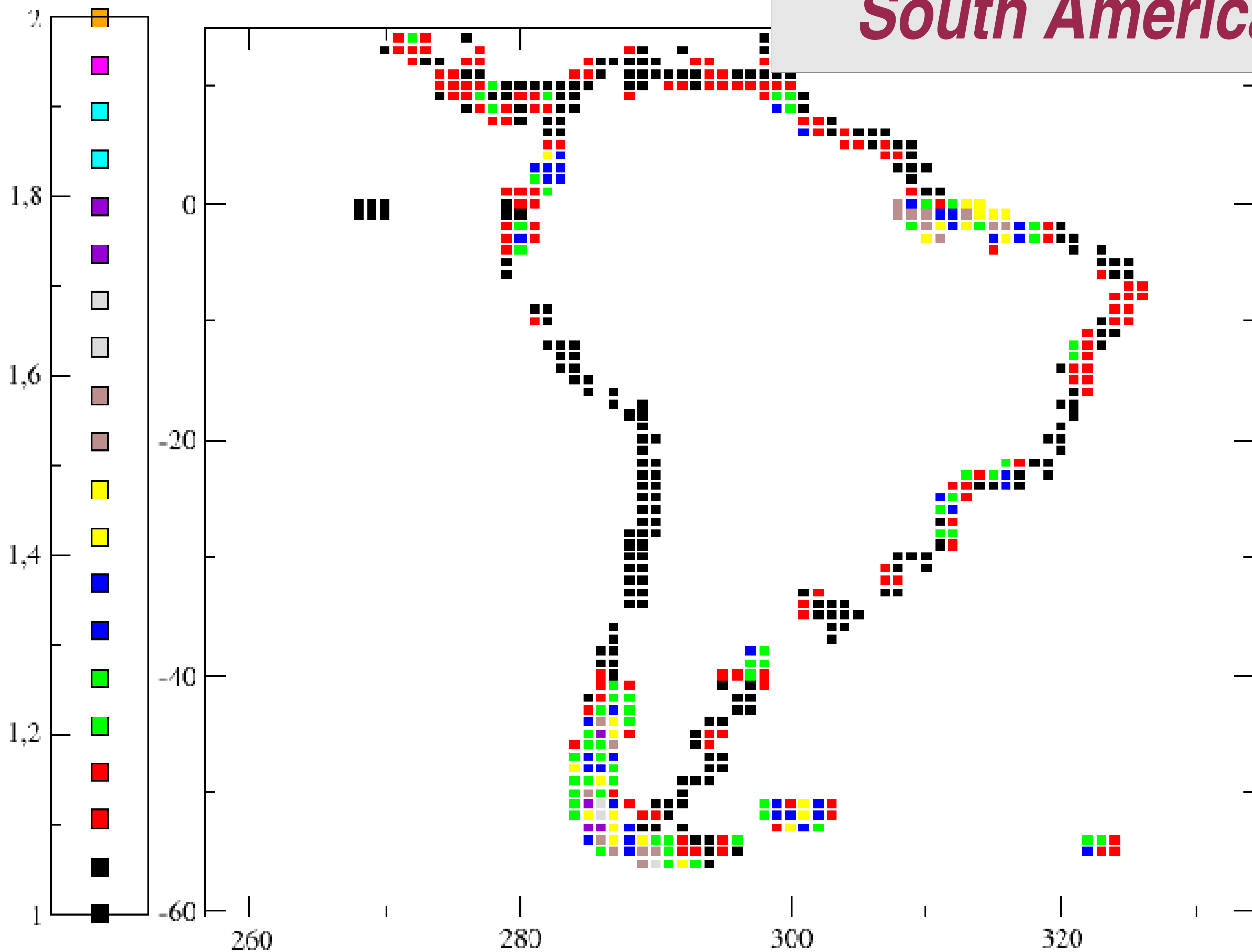
Africa



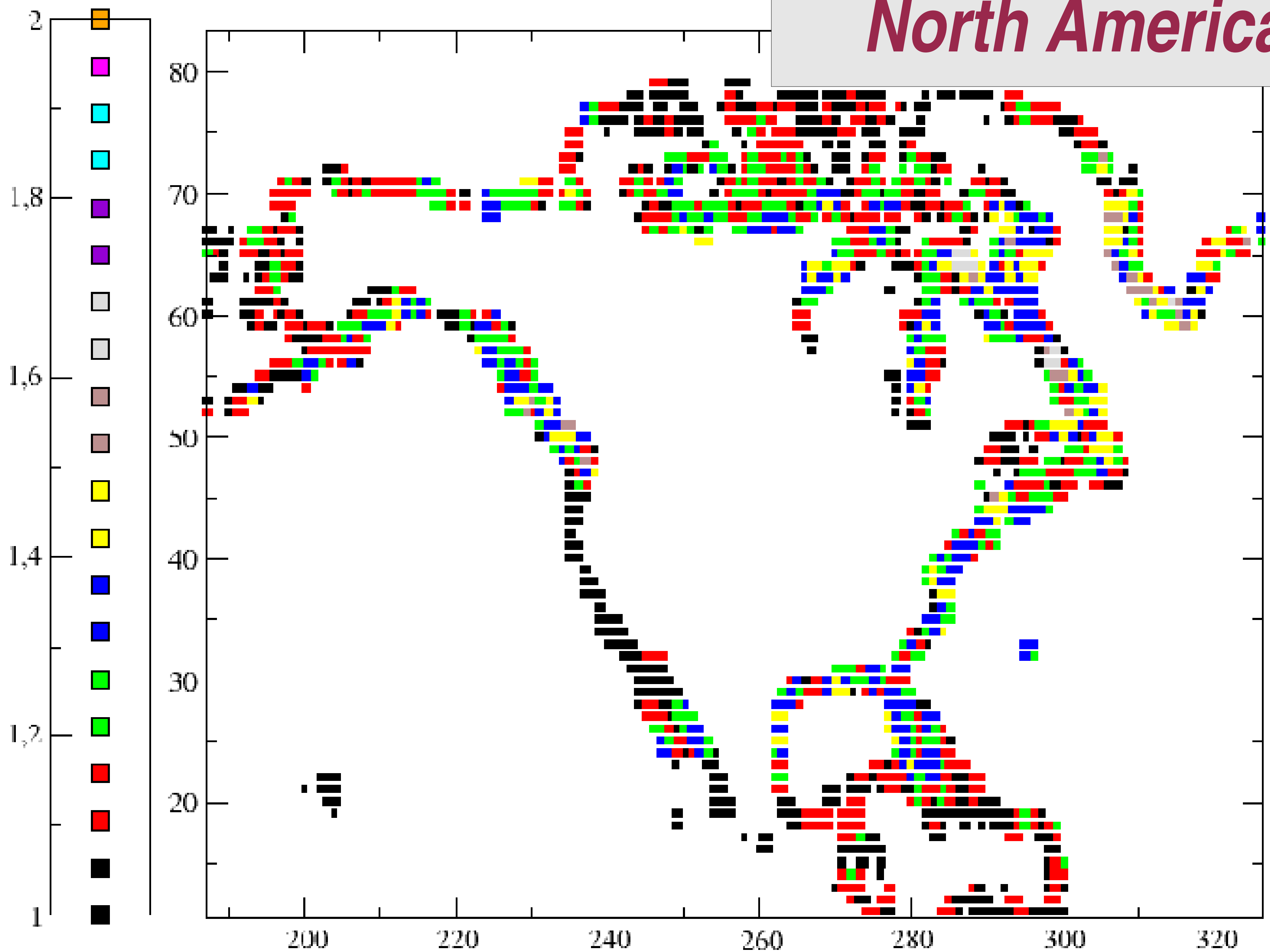
Oceania



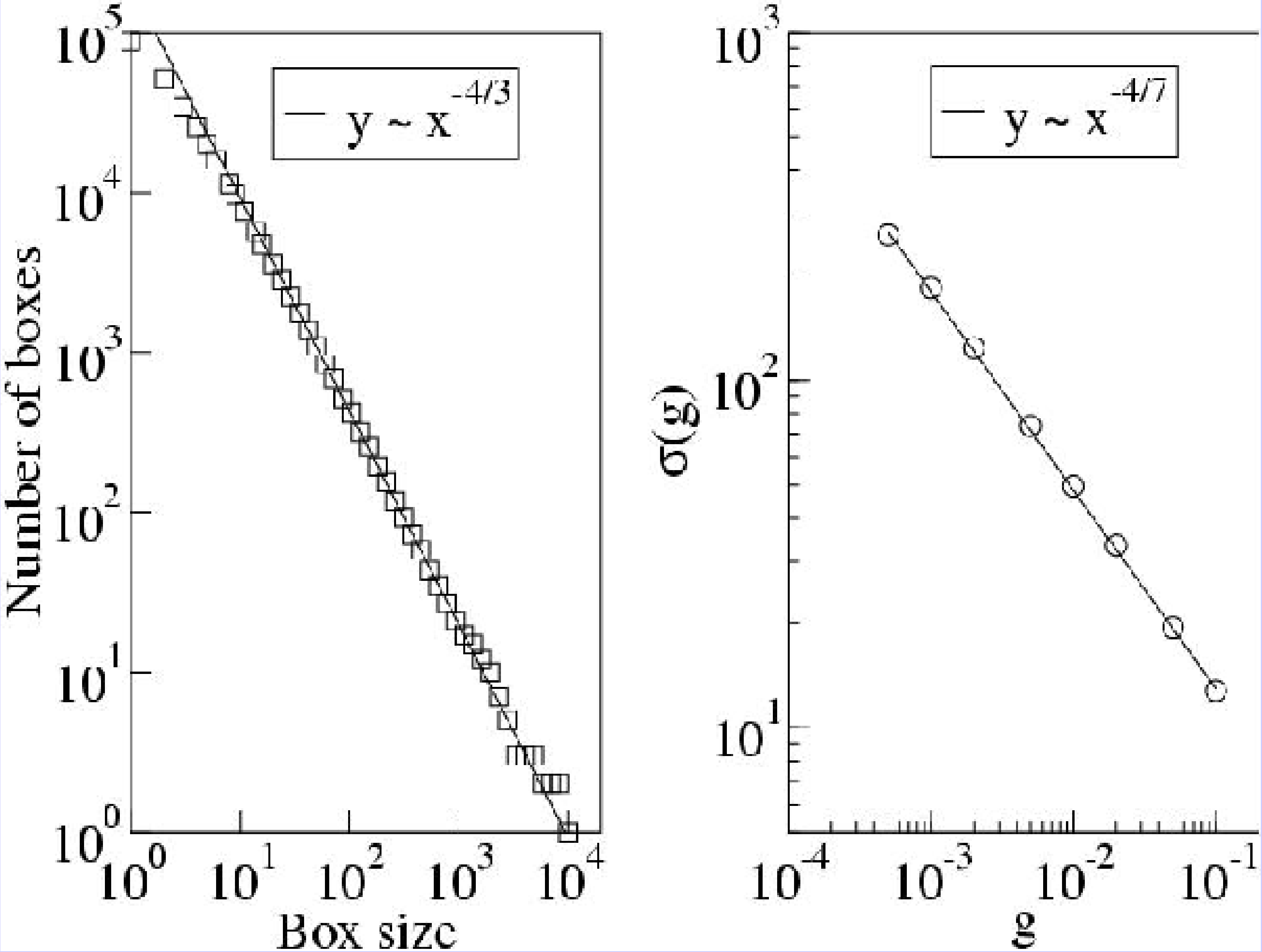
South America

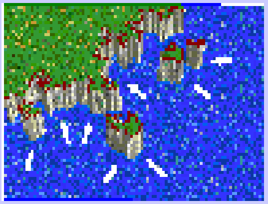


North America

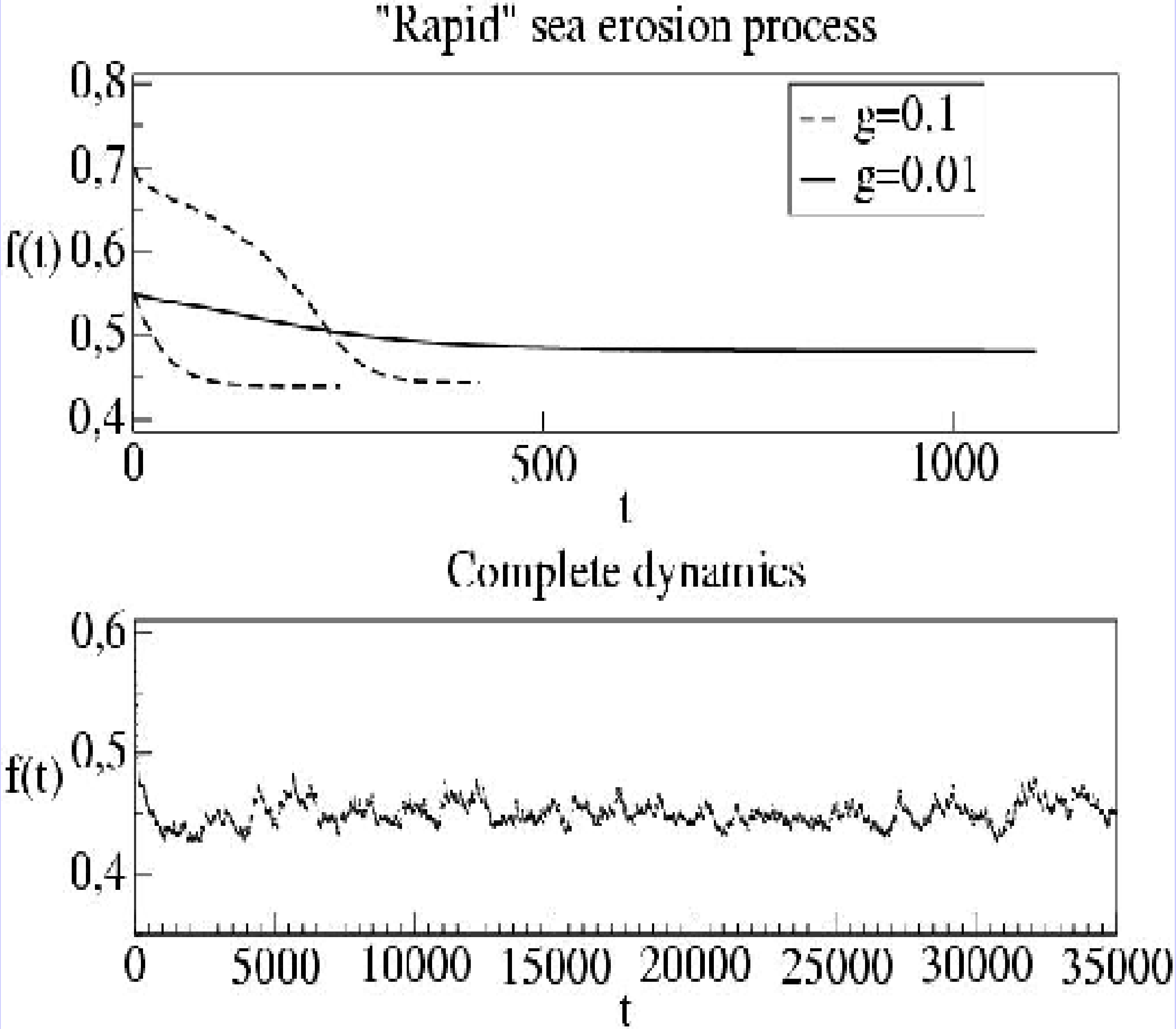


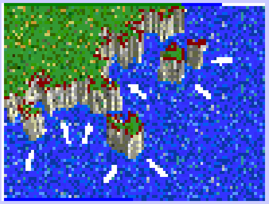
Model Universality class



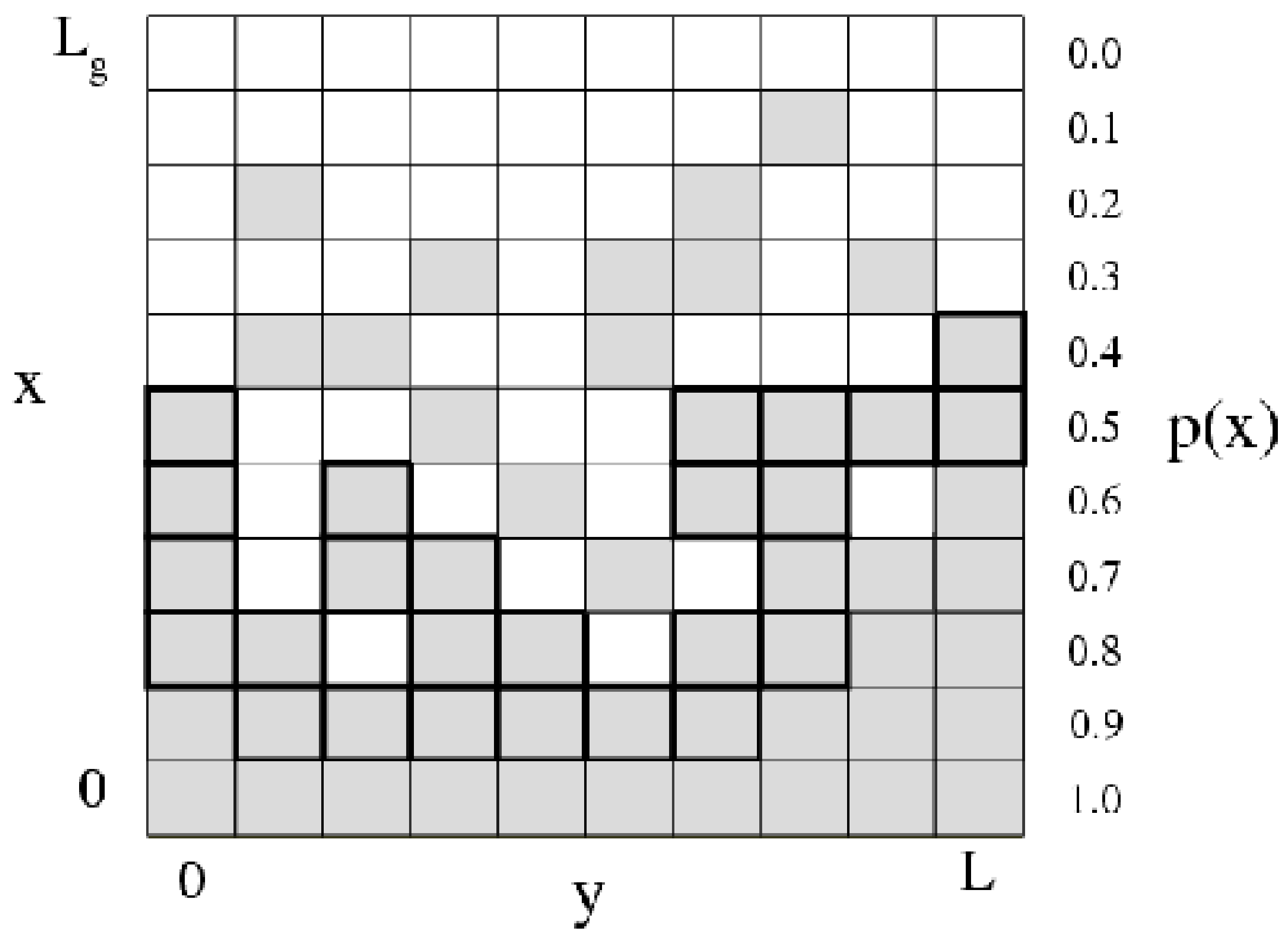


Model complete dynamics





Gradient percolation



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Links

The Coastline Extractor http://rimmer.ngdc.noaa.gov/coast/getcoast.html

Fractal Erosion http://www.forester.net/ecm_0205_fractal.html

Jean-Francois Colonna http://www.lactamme.polytechnique.fr/

Barton C., http://coastal.er.usgs.gov/barton/web

Greve C. http://www.usyd.edu.au/su/marine/lect/imsb/greve-l11.html

Barton C., http://coastal.er.usgs.gov/barton/pubs/fractalmap.pdf

Barton C., http://coastal.er.usgs.gov/barton/pubs/coastal.pdf

More materials and updates: http://greco.phys2.uniroma1.it/Coasts